Development and verification of computational methods for the study of soil-structure interaction — with applications to offshore wind-turbines

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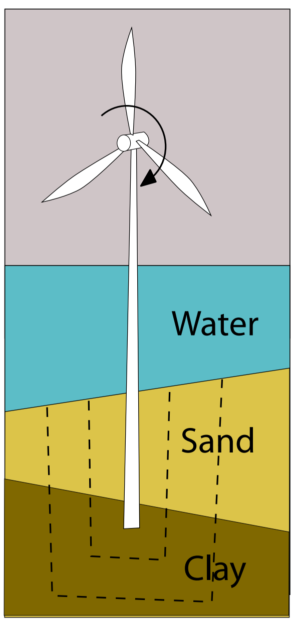
Introduction

Offshore wind-turbines are being constructed as an alternative to non-renewable energy sources. Energy production accounts for more than 82% of U.S. greenhouse gas emissions [1]. Coinciding with advances in turbine technology, the price of wind farms has been decreasing steadily [16]. The energy production using wind increased 3% from the year 2000 to 2001 in the U.S [2]. Still, wind power is more expensive than fossil-fuel power. Further advances in the technology of wind-power production are limited in part by the uncertainty associated with the mechanisms of soil-structure interaction. For many operational windmills, foundation construction and maintenance accounts for about one quarter of the cost of windmill power [16, 14]. Similarly, the power production capacity of a turbine is limited by the strength and reliability of the foundation [14].



Models of offshore constructions pose an additional computational difficulty since accurate and efficient models of wave propagation in saturated soil are not representable using standard finite and boundary element techniques. The cloning algorithm allows for the representation of an infinite medium using finite cells [8]. For the soil-structure interaction problem the cell element is necessarily concave. Discretizing the element into convex parts reduces the stability of the solution [13]. Newly developed two dimensional concave finite elements are being applied to the modeling of unbounded-media using the cloning algorithm. The approach shows promising results for two dimensional models. The three dimensional extension, necessary for the practical application, will require either a boundary element formulation or a three dimensional concave element.

At the Technical University Braunschweig in the *Institute A für Mechanik*, Prof. Heinz Antes and his colleagues have made significant advances in computational methods, specifically to the modeling of dynamic soil-structure interaction with boundary elements [12]. The efficacy of the proposed concave element cloning approach will be verified using his results. A comparison with the results produced by the dynamic soil-structure interaction using conventional scaled elements will also be performed [19].

Unlike available methods, the concave element approach can be applied irrespective of the material properties of the domain. If the results with respect to the boundary element formulations are reasonable, an extension to other types of soil models will be possible using the same concave element cloning model. Unbounded porous materials are of particular interest [17]. If successful, the computational model will provide a efficient alternative method for analyzing soil structure behavior. A first application will be the assessment of sites which are more suitable for new wind-turbine farms and improved windmill footing design.

1 Background research

General approaches developed for the approximation of soil-structure interaction behavior satisfy, at most, only one of the conditions governing the behavior of a semi-infinite domain: $^{(1)}$boundary displacements, or $^{(2)}$the field equation. Finite element methods, in particular, satisfy only the boundary displacement kinematic requirements of finite domains [7]. The extension of the finite element method to infinite domains

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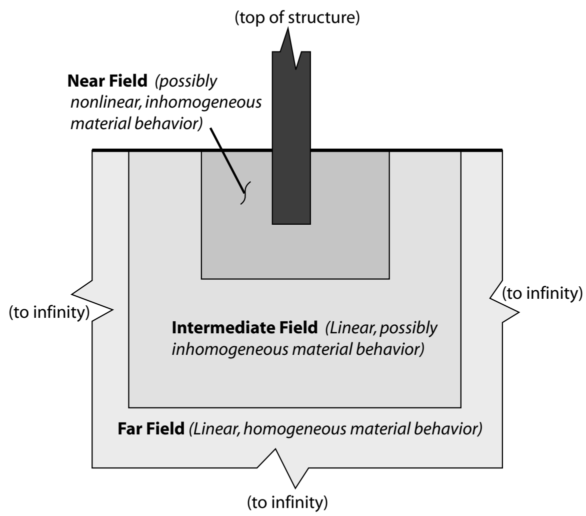
has resulted in approaches which either do not satisfy the far-field boundary condition (the Sommerfield Radiation condition) [4, 15]; or are coupled approaches based on a field equation solution [11, 6]. The accuracy and efficiency of such 'enriched' finite element methods has improved in the past 25 years [3]. Nevertheless, the built-in limitations often result in errors which overshadow the radiation damping components of the system stiffness matrix. Alternatively, the boundary element method utilizes test functions which satisfy the field equation exactly and requires only discretization of the near-field boundary not located at infinity [5]. For many field equations, the exact solution is either unknown or a higher order transcendental function which may be prohibitively expensive to calculate. The cloning-algorithm approach is analogous to the finite element approach on a finite domain. It satisfies all the boundary conditions, including the infinite. The method does not require a solution to the static or dynamic field equation [8, 18].



For the study of wind-turbine footings, the geometry of the footings is well-known but the material may be variable. Accordingly, a combination of the finite element and cloning algorithm approaches will be applied. The combination of approaches allows for a rigorous treatment of any non-linear behavior near the footing and the Sommerfield radiation condition in the far field. Current limitations of the cloning method include: instability of the resulting quadratic eigenvalue equation and requirement that the local and global stiffness matrices be proportional. For the two dimensional problem, using a concave shaped element will reduce the introduction of spurious modes into the model, allowing for a more efficient solution calculation [13]. Extending the cloning algorithm such that in-homogenous mediums can be represented in the intermediate field is the subject of ongoing research.

The analysis of the wind-turbine will also be conducted using boundary element methods and compared with results generated using absorbing boundaries and infinite elements. It is anticipated that the results from the cloning algorithm will be more robust than the enhanced finite element methods and more general than the boundary element method. The combination of two dimensional concave geometries and boundary element solutions may allow for an efficient three dimensional representation which can be employed in practical applications.

1.1 Concave element development

A concave shape formulation has been developed which satisfies the patch test, boundary conditions, and the ellipticity requirement while remaining differentiable and integrable within the concave domain. Given a set of shape functions $N_i(x, y)$ for a polygon Ω with n nodes, each node (i) located at the point $\{x_i, y_i\}$, the requirements can be restated as the patch test condition:

$$\textstyle\sum_{i=1}^{n} N_i(x, y) = 1, \quad \sum_{i=1}^{n} x_i N_i(x, y) = x \quad \text{and} \quad \sum_{i=1}^{n} y_i N_i(x, y) = y\,; \tag{1}$$

the boundedness or Chebycheff condition:

$$0 \leq N_i(x, y) \leq 1 \quad \forall (x, y) \in \Omega; \tag{2}$$

and the ellipticity or maximum-minimum principle

$$\mathcal{L}[N_i(x, y)] = 0 \quad \forall (x, y) \in \Omega, \quad \text{where } \mathcal{L}[\cdot] \text{ is an elliptic operator.} \tag{3}$$

For convenience the function $N_i(x, y)$ is unit valued at node (i) and zero at all other nodes.

The method for shape function construction, which is a computationally more efficient alternative to the Wachspress' derivation, can be applied to concave domains [10]. The derivation for a convex element depends on a function $r_{ij}(x, y)$ which is positive everywhere in the domain Ω except the boundary $\bar{ij}$ where it is zero. For a convex element the equation —which is zero between points (i) and (j)— can be linear. The determinant form is convenient:

$$\bar{r}_{ij}(x,y) = \begin{vmatrix} x & y & 1 \\ x_i & y_i & 1 \\ x_j & y_j & 1 \end{vmatrix} \tag{4}$$

According to the derivation, the $r_{ij}(x,y)$ are combined with constants to form a shape function $N_k(x,y)$. The function is zero along all boundaries except those adjacent to the named node (k) and satisfies the constancy requirement by construction, equation 1:

$$\phi_i(x,y) = \frac{k_k s_k(x,y)}{\sum_{l=1}^{n} k_n s_n(x,y)} \quad \text{where} \quad s_k(x,y) = \prod_{i \neq k \text{ and } i \neq k} r_{ij}(x,y) \tag{5}$$

If all the $r_{ij}(x,y)$ are strictly positive within the domain Ω then the denominator of the shape function is non zero within the domain.

For a concave element, if the function $r_{ij}(x,y)$ is linear, zero values are prescribed within the domain. Consequently a higher order representation must be used. The following function with elliptical contour lines can be used to describe right angled reentrant corners:

$$\hat{r}_{ij}(x,y) = \frac{1}{2x^2}\sqrt{\left(x^2-c^2\right)^2 + \left(c^2+x^2\right)y^2 + \left(x^2-c^2\right)\sqrt{\left(x^2+y^2\right)^2 + c^2\left(c^2+2\left(y^2-x^2\right)\right)}} \tag{6}$$

By translating and rotating the function $\hat{r}_{ij}(x,y)$ such that all the boundaries are described, a concave shape can be constructed. A right angled element with a right angled indentation can be employed as a cell in the cloning algorithm to solve a soil structure interaction problem.

Additional restrictions on $r_{ij}(x,y)$ include that it must be smooth over the domain and linear along the nodes adjacent to (i) and (j). If the restrictions are satisfied then the shape function is similarly linear along the edges and smooth in the interior of the domain.

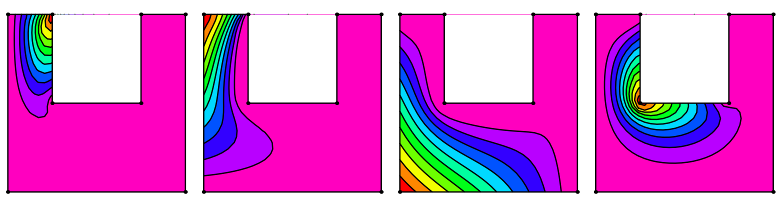


Figure 1: Interpolation contours within a concave cell

The example shape functions in figure 1 satisfy all of the requirements given in equations 1, 2 and **??** except the linearity requirements, equation 1. The linearity of the combination can be assured by adjusting the constants k_j. No part of the formulation requires that the k_j be constant, they need only be strictly positive within the domain. Solve for the positive functions using the following formulation:

$$\sum_{i=1}^{n}(x_i - x)k_i s_i = 0 \quad \text{and} \quad \sum_{i=1}^{n}(y_i - y)k_i s_i = 0 \tag{7}$$

The described method is applicable in general to any two dimensional domain with any number of side nodes [**?**]. No three dimensional analog exists as yet.

2 Methods

The cloning algorithm was conceived to generate static stiffness, mass and dynamic stiffness matrices for unbounded media [8, 9]. It exploits the likeness and similarity properties of static and dynamic stiffness matrices respectively to model an unbounded domain with a bounded element. Mass measures can similarly be added.

The static stiffness matrix is equal for two similarly shaped domains made of the same material. The dynamic stiffness of similar domains differ only by a constant. Exploiting this similarity the description of an unbounded media can be described in terms of the bounded media. For the static stiffness matrix case:

$$\begin{aligned}[\hat{K}]\vec{u}^1 &= [K^{II}]\vec{u}^1 + [K^{12}]\vec{u}^2 \\ -[\hat{K}]\vec{u}^2 &= [K^{21}]\vec{u}^1 + [K^{22}]\vec{u}^2\end{aligned} \tag{8}$$

Accordingly:

$$\begin{bmatrix} [K^{11}] - [\hat{K}] & [K^{12}] \\ [K^{21}] & [K^{22}] + [\hat{K}] \end{bmatrix} \begin{Bmatrix} \vec{u}^1 \\ \vec{u}^2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \tag{9}$$

Solving independently of the displacement $\vec{u}$:

$$\begin{aligned}(K^{11}] - [\hat{K}])([K^{22}] + [\hat{K}])^{\mathrm{T}} - [K^{12}][K^{21}]^{\mathrm{T}} &= 0 \\ [\hat{K}][\hat{K}]^{\mathrm{T}} + [\hat{K}][K^{22}]^{\mathrm{T}} - [K_{11}][\hat{K}]^{\mathrm{T}} + [K^{12}][K^{21}]^{\mathrm{T}} - [K^{11}][K^{22}]^{\mathrm{T}} &= 0\end{aligned} \tag{10}$$

Consequently, the derived finite concave cell can efficiently model an infinite media. Attempts using convex triangular and quadrilateral elements have failed in part due to the spurious waves generated between artificially generated sub-element boundaries. The derived concave element is infinitely smooth within the wave absorbing boundary.

If a fundamental solution, or Green's function solution, exists for a given material, the boundary element method can be applied [5]. It is not restricted to finite domains and can be used to efficiently model wave propagation behavior. Unfortunately, the fundamental solution with regard to soil media is either often a high order transcendental function or is not known. Nevertheless, the results of a cloning algorithm formulation using concave cells can be verified using a boundary element formulation for a linear-elastic homogeneous medium.

3 Time table

Given a twelve month schedule:

Time(months)	Task	Verify using
2	Concave element	Finite element soil – structure interaction results
3	Boundary element in 2D & 3D	Cloning and soil – structure interaction results
3	Combine concave and boundary element	3D Boundary element results
2	Implement developed method on real problem	Field results
2	Final report and journal papers	

The specific goals which will be completed along with the research are:

- a computer modeling program in which different wind mill footings can be tested
- A 3D hybrid element formulation employing 2D concave elements
- A 3D concave element formulation
- Windmill footing design criteria

Conclusion

The product of this research will be a method for representing the behavior of an unbounded media in three dimensions. The result will not only be useful to the study of windmill footings but to any unbounded problem including fluid structure interaction. For example: noise transmission through window panes; and material reliability, specifically crack propagation in crystalline materials could be investigated. The advancements in computational modeling achieved at *The Institute A für Mechanik* and computer algebra and shape function development will facilitate this new approach to generate approximate radiation conditions and efficiently model semi-infinite domains.

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